

# Superfield description of 10D SYM theory with magnetized extra dimensions

Hiroyuki Abe<sup>1,\*</sup>, Tatsuo Kobayashi<sup>2,†</sup>, Hiroshi Ohki<sup>3,‡</sup> and Keigo Sumita<sup>1,§</sup>

<sup>1</sup>*Department of Physics, Waseda University, Tokyo 169-8555, Japan*

<sup>2</sup>*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

<sup>3</sup>*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI),  
Nagoya University, Nagoya 464-8602, Japan*

## Abstract

We present a four-dimensional (4D)  $\mathcal{N} = 1$  superfield description of supersymmetric Yang-Mills (SYM) theory in ten-dimensional (10D) spacetime with certain magnetic fluxes in compactified extra dimensions preserving partial  $\mathcal{N} = 1$  supersymmetry out of full  $\mathcal{N} = 4$ . We derive a 4D effective action in  $\mathcal{N} = 1$  superspace directly from the 10D superfield action via dimensional reduction, and identify its dependence on dilaton and geometric moduli superfields. A concrete model for three generations of quark and lepton superfields are also shown. Our formulation would be useful for building various phenomenological models based on magnetized SYM theories or D-branes.

---

\*E-mail address: abe@waseda.jp

†E-mail address: kobayash@gauge.scphys.kyoto-u.ac.jp

‡E-mail address: ohki@kmi.nagoya-u.ac.jp

§E-mail address: k.sumita@moegi.waseda.jp

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>10D magnetized SYM in <math>\mathcal{N} = 1</math> superspace</b>	<b>2</b>
2.1	Toroidal compactification of 10D SYM theory . . . . .	2
2.2	Superfield description on nontrivial gauge background . . . . .	4
2.3	Zero-mode equations in superspace . . . . .	6
<b>3</b>	<b>4D effective action</b>	<b>6</b>
3.1	Dimensional reduction with magnetic fluxes . . . . .	7
3.2	4D effective action for zero-modes . . . . .	9
<b>4</b>	<b>Local supersymmetry and moduli multiplets</b>	<b>10</b>
4.1	4D $\mathcal{N} = 1$ effective supergravity . . . . .	11
4.2	Moduli dependence . . . . .	12
<b>5</b>	<b>An example of model building</b>	<b>14</b>
<b>6</b>	<b>Conclusion</b>	<b>16</b>
<b>A</b>	<b>Rescaling fields with some vanishing fluxes</b>	<b>17</b>

# 1 Introduction

Supersymmetric Yang-Mills (SYM) theories in higher-dimensional spacetime are quite attractive from both theoretical and phenomenological viewpoints. Such theories in ten-dimensional (10D) spacetime arise as low energy effective theories of superstrings, which are considered as the most promising candidates for an ultimate unification theory of elementary particles including gravitational interactions. It is indicated that SYM theories in various spacetime dimensions appear from D-branes in superstring theories. Various phenomenological superstring models beyond the standard model of elementary particles have been studied so far (for a review, see [1]), many of which are based on SYM effective theories. Even without mentioning superstring theories, SYM theories in higher-dimensional spacetime themselves are interesting enough for phenomenological model buildings beyond the standard model.

The standard model in four-dimensional (4D) spacetime may be realized at a low energy by compactifying the extra dimensions. In such a case, how to break higher-dimensional supersymmetry and to realize a chiral spectrum is the central issue. Toroidal compactifications with certain orbifold projections are mostly studied. Orbifolds may be considered as some singular limit of Calabi-Yau (CY) manifolds where most field contents, their wavefunctions and couplings in 4D effective theories are determined by geometric data. However, in general, such data are complicated and it is difficult to determine the CY metric explicitly, that makes phenomenological analyses on CY manifolds qualitative but not quantitative.

Chiral spectra can be obtained not only on such nontrivial geometric backgrounds, but also even on a simple toroidal background with some gauge fluxes in Yang-Mills (YM) sector. (see Ref. [2] and references therein).<sup>1</sup> In the latter case, chiral field contents and couplings are determined by the fluxes. For example, Yukawa couplings are determined by overlap integrals of the matter wavefunctions whose analytic forms are determined as functions of fluxes [2]. It is remarkable that the numbers of generations for chiral matter fields are determined by the fluxes, which provides a possibility that the flavor structure of the standard model is essentially determined by the YM-fluxes with sub-leading corrections from the background geometry. The observed flavor structures of quark and lepton masses and mixings would be realized by wavefunction localizations in extra dimensions due to the presence of magnetic fluxes [2, 4, 5] yielding some discrete flavor symmetries [6].<sup>2</sup> It is also remarkable that a certain class of magnetized D-branes can be regarded as the T-dual of some intersecting D-branes [2, 8].

Here we focus on such YM-flux backgrounds which preserve (at least) 4D  $\mathcal{N} = 1$  supersymmetry out of the full higher-dimensional supersymmetry such as  $\mathcal{N} = 4$ . In this case, field fluctuations around the background form  $\mathcal{N} = 1$  supermultiplets and their action is written by superfields in  $\mathcal{N} = 1$  superspace, as indicated in Refs. [9, 10] in the case without fluxes. In this paper we present a superfield description of SYM theories in higher-dimensional (especially 10D) spacetime with magnetic fluxes in extra dimensions preserving 4D  $\mathcal{N} = 1$  supersymmetry, and derive 4D effective action for zero-modes in the superspace. Such a superfield description allows us to construct phenomenological models systematically and to analyze detailed low-energy properties of them, such as particle and superparticle flavor structures, features of Higgs par-

---

<sup>1</sup> Other geometrical backgrounds with magnetic fluxes were also studied (see e.g. [3]).

<sup>2</sup> Similar non-Abelian discrete flavor symmetries are obtained in heterotic orbifold models [7].

ticles and dark matter candidates, which are required for verifying these models by recent and upcoming data obtained from high-energy experiments as well as cosmological observations.

The sections are organized as follows. In Sec. 2, we rewrite the 10D SYM action in 4D  $\mathcal{N} = 1$  superspace in the case with magnetic fluxes in extra dimensions, and derive zero-mode equations for superfields. In Sec. 3, the 4D effective action for zero-modes are shown after a dimensional reduction. The action is extended to the case with local supersymmetry (i.e. supergravity) in Sec. 4 where its moduli dependence is also identified. A direction of model building based on the effective action is indicated in Sec. 5. Finally, Sec. 6 is devoted to conclusions.

## 2 10D magnetized SYM in $\mathcal{N} = 1$ superspace

In this section, we briefly review a compactification of 10D SYM theory on flat 4D Minkowski spacetime times a product of factorizable three tori  $T^2 \times T^2 \times T^2$ . Then we derive a superfield description suitable for such a compactification with certain magnetic fluxes preserving 4D  $\mathcal{N} = 1$  supersymmetry. The geometric (torus) parameter dependence is explicitly shown in this procedure, which is important to determine couplings between YM and moduli superfields in the 4D effective action for zero-modes derived in the next sections.

### 2.1 Toroidal compactification of 10D SYM theory

The action of 10D SYM theory is given by

$$S = \int d^{10}X \sqrt{-G} \frac{1}{g^2} \text{Tr} \left[ -\frac{1}{4} F^{MN} F_{MN} + \frac{i}{2} \bar{\lambda} \Gamma^M D_M \lambda \right], \quad (1)$$

where  $g$  is a 10D YM gauge coupling constant and the trace runs over the adjoint representation of a gauge group. The 10D spacetime coordinates are expressed by  $X^M$ , and the vector/tensor indices  $M, N = 0, 1, \dots, 9$  are lowered and raised by the 10D metric  $G_{MN}$  and its inverse  $G^{MN}$ , respectively. The YM field strength  $F_{MN}$  and the covariant derivative  $D_M$  are defined by

$$\begin{aligned} F_{MN} &= \partial_M A_N - \partial_N A_M - i[A_M, A_N], \\ D_M \lambda &= \partial_M \lambda - i[A_M, \lambda], \end{aligned}$$

for a 10D vector (gauge) field  $A_M$  and a 10D Majorana-Weyl spinor field  $\lambda$  satisfying  $\lambda^C = \lambda$  and  $\Gamma \lambda = +\lambda$  where  $\lambda^C$  is a 10D charge conjugation of  $\lambda$ , and  $\Gamma$  is a 10D chirality operator.

We decompose the 10D real coordinates  $X^M = (x^\mu, y^m)$  into 4D Minkowski spacetime coordinates  $x^\mu$  with  $\mu = 0, 1, 2, 3$  and six dimensional (6D) extra space coordinates  $y^m$  with  $m = 4, \dots, 9$ . Note that  $\mu = 0$  describes the time component. Similarly the 10D vector field is decomposed as  $A_M = (A_\mu, A_m)$ . Our convention of the background metric is chosen as

$$ds^2 = G_{NN} dX^M dX^N = \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n,$$

where  $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ . Then, we consider a torus compactification of internal 6D space  $y^m$  by identifying  $y^m \sim y^m + 2$ . The 6D torus is further assumed as a product of factorizable three tori,  $T^2 \times T^2 \times T^2$ , and then the extra 6D metric can be described as

$$g_{mn} = \begin{pmatrix} g^{(1)} & 0 & 0 \\ 0 & g^{(2)} & 0 \\ 0 & 0 & g^{(3)} \end{pmatrix},$$

where each of entries is a  $2 \times 2$  matrix and submatrices in the diagonal part are given by

$$g^{(i)} = (2\pi R_i)^2 \begin{pmatrix} 1 & \text{Re } \tau_i \\ \text{Re } \tau_i & |\tau_i|^2 \end{pmatrix}, \quad (2)$$

for  $i = 1, 2, 3$ . The real and complex parameters  $R_i$  and  $\tau_i$ , respectively, determine the size and the shape of  $i$ th torus  $T^2$ . Especially the area  $\mathcal{A}^{(i)}$  of the  $i$ th torus is described as

$$\mathcal{A}^{(i)} = (2\pi R_i)^2 \text{Im } \tau_i.$$

For convenience in a supersymmetric world, we introduce complex (extra-dimensional) coordinates  $z^i$  for  $i = 1, 2, 3$  and also complex vector components  $A_i$  defined by

$$\begin{aligned} z^i &\equiv \frac{1}{2}(y^{2+2i} + \tau_i y^{3+2i}), & \bar{z}^{\bar{i}} &\equiv (z^i)^*, \\ A_i &\equiv -\frac{1}{\text{Im } \tau_i}(\tau_i^* A_{2+2i} - A_{3+2i}), & \bar{A}_{\bar{i}} &\equiv (A_i)^\dagger. \end{aligned} \quad (3)$$

The torus boundary conditions are given by  $z^i \sim z^i + 1$  and  $z^i \sim z^i + \tau^i$ . A metric  $h_{i\bar{j}}$  for the complex coordinates is extracted from

$$ds_{6D}^2 = g_{mn} dy^m dy^n \equiv 2h_{i\bar{j}} dz^i d\bar{z}^{\bar{j}},$$

and then we find

$$h_{i\bar{j}} = 2(2\pi R_i)^2 \delta_{i\bar{j}} = \delta_{i\bar{j}} e_i^{\bar{i}} \bar{e}_{\bar{j}}^{\bar{j}},$$

where  $e_i^{\bar{i}}$  is a vielbein defined by

$$e_i^{\bar{i}} = \sqrt{2}(2\pi R_i) \delta_i^{\bar{i}},$$

with its inverse  $e_i^{\bar{i}}$  and complex conjugate  $\bar{e}_{\bar{i}}^{\bar{i}}$  satisfying

$$e_i^{\bar{i}} e_i^{\bar{j}} = \delta_i^{\bar{j}}, \quad e_i^{\bar{i}} e_i^{\bar{j}} = \delta_i^{\bar{j}}, \quad \bar{e}_{\bar{i}}^{\bar{i}} = (e_i^{\bar{i}})^*,$$

and Roman indices representing local Lorentz space. The *Italic* (Roman) indices  $i, j, \dots$  ( $\bar{i}, \bar{j}, \dots$ ) are lowered and raised by the metric  $h_{i\bar{j}}$  and its inverse  $h^{\bar{i}j}$  ( $\delta_{i\bar{j}}$  and its inverse  $\delta^{\bar{i}j}$ ), respectively.

## 2.2 Superfield description on nontrivial gauge background

The 10D SYM theory has  $\mathcal{N} = 4$  supersymmetry in terms of a 4D supercharge. The YM fields  $A_M$  and  $\lambda$  can be decomposed into 4D  $\mathcal{N} = 1$  (on-shell) supermultiplets as

$$\mathbf{V} = \{A_\mu, \lambda_0\}, \quad \boldsymbol{\phi}_i = \{A_i, \lambda_i\}.$$

Here the 10D Majorana-Weyl spinor  $\lambda$  is decomposed into four 4D Weyl (or equivalently Majorana) spinors  $\lambda_0$  and  $\lambda_i$  satisfying

$$\Gamma^{(i)}\lambda_0 = +\lambda_0, \quad \Gamma^{(i)}\lambda_j = +\lambda_j \quad (i=j), \quad \Gamma^{(i)}\lambda_j = -\lambda_j \quad (i \neq j), \quad (4)$$

where  $\Gamma^{(i)}$  for each  $i = 1, 2, 3$  are a chirality operator associated with 6D spacetime coordinates  $(x^\mu, z^i)$ . Note that an eigenvalue of the 10D chirality operator  $\Gamma$  is obtained by a product of the eigenvalues of  $\Gamma^{(1)}$ ,  $\Gamma^{(2)}$  and  $\Gamma^{(3)}$ . If we write the chirality of  $i$ th complex coordinate  $z_i$  in the  $i$ th subscript of  $\lambda$  like  $\lambda_{\pm\pm\pm}$ , the decomposed spinor fields  $\lambda_0$  and  $\lambda_i$  are identified with the chirality eigenstate  $\lambda_{\pm\pm\pm}$  as

$$\lambda_0 = \lambda_{+++}, \quad \lambda_1 = \lambda_{+--}, \quad \lambda_2 = \lambda_{-+-}, \quad \lambda_3 = \lambda_{--+}.$$

Note that the components  $\lambda_{---}$ ,  $\lambda_{-++}$ ,  $\lambda_{+-+}$  and  $\lambda_{++-}$  do not exist in the 10D Majorana-Weyl spinor  $\lambda$  due to the condition  $\Gamma\lambda = +\lambda$ .

The above  $\mathcal{N} = 1$  vector multiplet  $\mathbf{V}$  and chiral multiplets  $\boldsymbol{\phi}_i$  are expressed, respectively, by a vector superfield  $V$  and a chiral superfield  $\phi_i$ . Our definition of superfields is as follows:

$$\begin{aligned} V &\equiv -\theta\sigma^\mu\bar{\theta}A_\mu + i\bar{\theta}\bar{\theta}\theta\lambda_0 - i\theta\theta\bar{\theta}\bar{\lambda}_0 + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D, \\ \phi_i &\equiv \frac{1}{\sqrt{2}}A_i + \sqrt{2}\theta\lambda_i + \theta\theta F_i, \end{aligned}$$

where  $\theta$  and  $\bar{\theta}$  are Grassmann coordinates of 4D  $\mathcal{N} = 1$  superspace with spinor indices  $\alpha$  and  $\dot{\alpha}$  omitted respectively<sup>3</sup>. The superfield description allows us to express the 10D SYM action (1) in  $\mathcal{N} = 1$  superspace as [10]

$$S = \int d^{10}X \sqrt{-G} \left[ \int d^4\theta \mathcal{K} + \left\{ \int d^2\theta \left( \frac{1}{4g^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \mathcal{W} \right) + \text{h.c.} \right\} \right], \quad (5)$$

where

$$\begin{aligned} \mathcal{K} &= \frac{2}{g^2} h^{\bar{i}j} \text{Tr} \left[ \left( \sqrt{2}\bar{\partial}_{\bar{i}} + \bar{\phi}_{\bar{i}} \right) e^{-V} \left( -\sqrt{2}\partial_j + \phi_j \right) e^V + \bar{\partial}_{\bar{i}} e^{-V} \partial_j e^V \right] + \mathcal{K}_{\text{WZW}}, \\ \mathcal{W} &= \frac{1}{g^2} \epsilon^{\text{ijk}} e_i{}^{\dot{i}} e_j{}^{\dot{j}} e_k{}^{\dot{k}} \text{Tr} \left[ \sqrt{2} \phi_i \left( \partial_j \phi_k - \frac{1}{3\sqrt{2}} [\phi_j, \phi_k] \right) \right], \end{aligned}$$

with a totally antisymmetric tensor  $\epsilon^{\text{ijk}}$  and  $\epsilon^{123} = 1$ . The field strength superfield  $\mathcal{W}_\alpha$  is defined by  $\mathcal{W}_\alpha \equiv -\frac{1}{4} \bar{D} \bar{D} e^{-V} D_\alpha e^V$  where  $D_\alpha$  and  $\bar{D}_{\dot{\alpha}}$  are a supercovariant derivative and its

---

<sup>3</sup>Our conventions mostly follow those in Ref. [11].

conjugate with 4D spinor indices  $\alpha$  and  $\dot{\alpha}$ , respectively. The term  $\mathcal{K}_{\text{WZW}}$  corresponds to a Wess-Zumino-Witten term which vanishes in Wess-Zumino (WZ) gauge. The equations of motion for auxiliary fields  $D$  and  $F_i$  lead to

$$D = -h^{\bar{i}j} \left( \bar{\partial}_{\bar{i}} A_j + \partial_j \bar{A}_{\bar{i}} + \frac{1}{2} [\bar{A}_{\bar{i}}, A_j] \right), \quad (6)$$

$$\bar{F}_{\bar{i}} = -h_{j\bar{i}} \epsilon^{\text{jkl}} e_j^j e_k^k e_l^l \left( \partial_k A_l - \frac{1}{4} [A_k, A_l] \right). \quad (7)$$

The original SYM action (1) is obtained after integrating over the superspace in Eq. (5) and substituting on-shell values (6) and (7).

We assume 4D Lorentz invariant and (at least  $\mathcal{N} = 1$ ) supersymmetric VEVs of fields,

$$\langle A_i \rangle \neq 0, \quad \langle A_{\mu} \rangle = \langle \lambda_0 \rangle = \langle \lambda_i \rangle = \langle F_i \rangle = \langle D \rangle = 0. \quad (8)$$

We will see later<sup>4</sup> that certain magnetized background satisfy Eq. (8). Then we extract fluctuations  $\tilde{V}$  and  $\tilde{\phi}_i$  of superfields  $V$  and  $\phi_i$ , respectively, around the vacuum configuration (8) as

$$V \equiv \langle V \rangle + \tilde{V}, \quad \phi_i \equiv \langle \phi_i \rangle + \tilde{\phi}_i,$$

where  $\langle V \rangle = 0$  and  $\langle \phi_i \rangle = \langle A_i \rangle / \sqrt{2}$  due to Eq. (8). For a notational convenience, in the following, we omit tildes and use original notation  $V$  and  $\phi_i$  for their corresponding fluctuations around the vacuum. Then, in the WZ-gauge, the functions  $\mathcal{K}$  and  $\mathcal{W}$  are expanded in powers of  $V$  as

$$\begin{aligned} \mathcal{K} = & \frac{2}{g^2} h^{\bar{i}j} \text{Tr} \left[ \bar{\phi}_{\bar{i}} \phi_j + \sqrt{2} \left\{ \left( \bar{\partial}_{\bar{i}} \phi_j + \frac{1}{\sqrt{2}} [\langle \bar{\phi}_{\bar{i}} \rangle, \phi_j] + \text{h.c.} \right) + \frac{1}{\sqrt{2}} [\bar{\phi}_{\bar{i}}, \phi_j] \right\} V \right. \\ & \left. + (\bar{\partial}_{\bar{i}} V)(\partial_j V) + \frac{1}{2} (\bar{\phi}_{\bar{i}} \phi_j + \phi_j \bar{\phi}_{\bar{i}}) V^2 - \bar{\phi}_{\bar{i}} V \phi_j V \right] + \mathcal{K}^{(\text{D})} + \mathcal{K}^{(\text{br})}, \end{aligned} \quad (9)$$

$$\mathcal{W} = \frac{1}{g^2} \epsilon^{\text{ijk}} e_i^i e_j^j e_k^k \text{Tr} \left[ \sqrt{2} \left( \partial_i \phi_j - \frac{1}{\sqrt{2}} [\langle \phi_i \rangle, \phi_j] \right) \phi_k - \frac{2}{3} \phi_i \phi_j \phi_k \right] + \mathcal{W}^{(\text{F})}, \quad (10)$$

where

$$\begin{aligned} \mathcal{K}^{(\text{br})} = & \frac{1}{g^2} h^{\bar{i}j} \text{Tr} \left[ \frac{1}{2} (\langle \bar{\phi}_{\bar{i}} \rangle \langle \phi_j \rangle + \langle \phi_j \rangle \langle \bar{\phi}_{\bar{i}} \rangle \langle \bar{\phi}_{\bar{i}} \rangle \phi_j + \phi_j \langle \bar{\phi}_{\bar{i}} \rangle + \bar{\phi}_{\bar{i}} \langle \phi_j \rangle + \langle \phi_j \rangle \bar{\phi}_{\bar{i}}) V^2 \right. \\ & \left. - \langle \bar{\phi}_{\bar{i}} \rangle V \langle \phi_j \rangle V - \langle \bar{\phi}_{\bar{i}} \rangle V \phi_j V - \bar{\phi}_{\bar{i}} V \langle \phi_j \rangle V \right], \end{aligned} \quad (11)$$

and we omit terms in higher-powers of  $\theta$  and  $\bar{\theta}$  than  $\theta^2 \bar{\theta}^2$  which vanish in the superspace action (5). The two terms of  $\mathcal{K}^{(\text{D})}$  and  $\mathcal{W}^{(\text{F})}$  are eliminated in the case that the SUSY conditions (6) and (7) are satisfied. The vacuum configuration (8) in general breaks the gauge symmetry (as well as the higher-dimensional supersymmetry) of original SYM theory down to its subgroup, and vector multiplets associated with the broken symmetries become massive. Such masses are generated in  $\mathcal{K}^{(\text{br})}$ .

---

<sup>4</sup>See Eqs.(18), (19) and (48).

### 2.3 Zero-mode equations in superspace

On the  $\mathcal{N} = 1$  supersymmetric background (8), Kaluza-Klein (KK) mode-equations should be written as superfield equations. Then, mode expansions of superfields  $V$  and  $\phi_j$  are expressed as

$$V(x^\mu, \mathbf{z}, \bar{\mathbf{z}}) = \sum_{\mathbf{n}} \left( \prod_i f_0^{(i), n_i}(z^i, \bar{z}^{\bar{i}}) \right) V^{\mathbf{n}}(x^\mu), \quad (12)$$

$$\phi_j(x^\mu, \mathbf{z}, \bar{\mathbf{z}}) = \sum_{\mathbf{n}} \left( \prod_i f_j^{(i), n_i}(z^i, \bar{z}^{\bar{i}}) \right) \phi_j^{\mathbf{n}}(x^\mu), \quad (13)$$

where  $\mathbf{z} = (z^1, z^2, z^3)$ ,  $\bar{\mathbf{z}} = (\bar{z}^{\bar{1}}, \bar{z}^{\bar{2}}, \bar{z}^{\bar{3}})$ ,  $\mathbf{n} = (n_1, n_2, n_3)$  and  $n_i \in \mathbf{Z}$  for  $i, j = 1, 2, 3$ . The 4D fields  $V^{\mathbf{n}}$  and  $\phi_j^{\mathbf{n}}$  are KK modes of 10D fields  $V$  and  $\phi_j$ , respectively, with KK momenta of three tori labeled by  $\mathbf{n}$ . The functions  $f_0^{(i), n_i}$  and  $f_j^{(i), n_i}$  are wavefunctions of  $V^{\mathbf{n}}$  and  $\phi_j^{\mathbf{n}}$ , respectively, in the  $i$ th torus. Note that all the quantities  $\{V, \phi_j, V^{\mathbf{n}}, \phi_j^{\mathbf{n}}, f_0^{(i), n_i}, f_j^{(i), n_i}\} = \mathcal{G}$  in Eqs. (12) and (13) have the same subscripts of YM indices implicitly, e.g.,  $\mathcal{G} = \mathcal{G}_{AB}$  with  $A, B = 1, 2, \dots, N$  for  $U(N)$  SYM theory, because the both-hand sides of Eqs. (12) and (13) are adjoint matrices of YM gauge group, and wavefunctions can be different from each other element by element due to a possible gauge symmetry breaking caused by  $\langle A_i \rangle \neq 0$  in Eq. (8). We should remark that  $V^{\mathbf{n}}$  and  $\phi_j^{\mathbf{n}}$  are a vector and chiral superfields, respectively, while  $f_0^{(i), n_i}$  and  $f_j^{(i), n_i}$  are independent of superspace coordinates  $\theta$  and  $\bar{\theta}$  on a  $\mathcal{N} = 1$  supersymmetric background (8).

Substituting Eqs. (12) and (13) into the superspace action (5) with Eqs. (9) and (10), we find zero-mode equations as

$$\begin{aligned} \partial_i f_0^{(i), n_i=0} - \frac{1}{\sqrt{2}} \left[ \langle \phi_i \rangle, f_0^{(i), n_i=0} \right] &= 0, \\ \bar{\partial}_{\bar{i}} f_j^{(i), n_i=0} + \frac{1}{\sqrt{2}} \left[ \langle \bar{\phi}_{\bar{i}} \rangle, f_j^{(i), n_i=0} \right] &= 0 \quad (i = j), \end{aligned} \quad (14)$$

$$\partial_i f_j^{(i), n_i=0} - \frac{1}{\sqrt{2}} \left[ \langle \phi_i \rangle, f_j^{(i), n_i=0} \right] = 0 \quad (i \neq j), \quad (15)$$

with which terms quadratic in superfields  $\phi_i$  and  $V$  are eliminated in Eqs. (9) and (10) except in Eq. (11), and then vanishing masses for zero-modes  $\phi_j^{\mathbf{n}=0}$  are guaranteed. Note that a sign-difference between the second terms in the left-hand sides of Eqs. (14) and (15) comes from the chirality structure (4). A remarkable point is that zero-mode equations are written for superfields. Such a superfield description allows us to perform a dimensional reduction while keeping a manifest  $\mathcal{N} = 1$  superspace structure preserved by the gauge background (8).

## 3 4D effective action

In the following, we focus on 4D massless modes (called zero-modes) in chiral superfields  $\phi_i$  which possess nontrivial wavefunction profiles under the existence of magnetic fluxes in extra



dimensions. We derive a 4D effective action for zero-modes by solving the zero-mode equations and performing integrations over extra space coordinates in the 10D action, while keeping the  $\mathcal{N} = 1$  superspace structure preserved by the flux background.

### 3.1 Dimensional reduction with magnetic fluxes

Because we focus on zero-modes in the following, for a notational convenience, their wavefunctions are denoted as

$$f_0^{(i),n_i=0} \equiv f_0^{(i)}, \quad f_j^{(i),n_i=0} \equiv f_j^{(i)}.$$

After substituting  $\langle \phi_i \rangle = \langle A_i \rangle / \sqrt{2}$  into Eqs. (14) and (15), zero-mode equations are expressed by

$$\bar{\partial}_{\bar{i}} f_j^{(i)} + \frac{1}{2} [\langle \bar{A}_{\bar{i}} \rangle, f_j^{(i)}] = 0 \quad (i = j), \quad (16)$$

$$\partial_i f_j^{(i)} - \frac{1}{2} [\langle A_i \rangle, f_j^{(i)}] = 0 \quad (i \neq j). \quad (17)$$

It is found that Eqs.(16) and (17) are equivalent to zero-mode equations for charged fermion fields  $\lambda_j$  in the gauge background [2] as it should be.

In the following we consider the case that the YM gauge group is  $U(N)$  and assume a supersymmetric (Abelian) magnetic background:

$$\langle A_i \rangle = \frac{\pi}{\text{Im } \tau_i} (M^{(i)} \bar{z}_i + \bar{\zeta}_i),$$

where

$$M^{(i)} = \text{diag}(M_1^{(i)}, M_2^{(i)}, \dots, M_N^{(i)}),$$

$$\zeta_i = \text{diag}(\zeta_1^{(i)}, \zeta_2^{(i)}, \dots, \zeta_N^{(i)}),$$

are  $N \times N$  diagonal matrices of Abelian magnetic fluxes and Wilson-lines, respectively. Here, the magnetic fluxes must satisfy  $M_1^{(i)}, M_2^{(i)}, \dots, M_N^{(i)} \in \mathbf{Z}$  due to the Dirac's quantization condition. We assume that the above magnetic fluxes are further constrained in such a way that they satisfy supersymmetry condition

$$h^{\bar{i}j} (\bar{\partial}_{\bar{i}} \langle A_j \rangle + \partial_j \langle \bar{A}_{\bar{i}} \rangle) = 0, \quad (18)$$

$$\epsilon^{jkl} e_k^j e_l^i \partial_k \langle A_l \rangle = 0, \quad (19)$$

in order to satisfy Eq. (8) with Eqs. (6) and (7) for the Abelian background.

In the case that all the magnetic fluxes  $M_1^{(i)}, M_2^{(i)}, \dots, M_N^{(i)}$  take different values from each other, the YM gauge symmetry is broken down as  $U(N) \rightarrow U(1)^N$  by the existence of fluxes. On the other hand, if any of them degenerate, e.g.,

$$M_1^{(i)} = M_2^{(i)} = \dots = M_{N_1}^{(i)},$$

$$M_{N_1+1}^{(i)} = M_{N_1+2}^{(i)} = \dots = M_{N_1+N_2}^{(i)},$$

$$\vdots$$

$$M_{N_1+N_2+\dots+N_{\tilde{N}-1}+1}^{(i)} = M_{N_1+N_2+\dots+N_{\tilde{N}-1}+2}^{(i)} = \dots = M_{N_1+N_2+\dots+N_{\tilde{N}-1}+N_{\tilde{N}}}^{(i)}, \quad (20)$$

with  $\sum_a N_a = N$  and  $M_{N_a} \neq M_{N_b}$  for  $a, b = 1, 2, \dots, \tilde{N}$  and  $a \neq b$ , the breaking pattern is changed as  $U(N) \rightarrow \prod_a U(N_a)$ . The same holds for Wilson-lines. In the following, indices  $a, b, c = 1, 2, \dots, \tilde{N}$  label the unbroken YM subgroups on the flux and Wilson-line background, and traces in expressions are performed within such subgroups.

From Eqs. (16) and (17), for the unbroken YM subgroup labeled by  $a$  and  $b$ , we find zero-mode equations  $(f_j^{(i)})_{ab}$  as

$$\left[ \bar{\partial}_i + \frac{\pi}{2\text{Im } \tau_i} \left( M_{ab}^{(i)} z_i + \zeta_{ab}^{(i)} \right) \right] (f_j^{(i)})_{ab} = 0 \quad (i = j), \quad (21)$$

$$\left[ \partial_i - \frac{\pi}{2\text{Im } \tau_i} \left( M_{ab}^{(i)} \bar{z}_i + \bar{\zeta}_{ab}^{(i)} \right) \right] (f_j^{(i)})_{ab} = 0 \quad (i \neq j), \quad (22)$$

where

$$M_{ab}^{(i)} = M_{N_a}^{(i)} - M_{N_b}^{(i)}, \quad \zeta_{ab}^{(i)} = \zeta_{N_a}^{(i)} - \zeta_{N_b}^{(i)}.$$

For  $i = j$ , a normalizable solution of Eq. (21) is found as

$$(f_j^{(i)})_{ab} = f_{ab}^{I^{(i)}} \equiv \begin{cases} \Theta^{I_{ab}^{(i)}, M_{ab}^{(i)}}(\tilde{z}_i) & (M_{ab}^{(i)} > 0) \\ (\mathcal{A}^{(i)})^{-1/2} & (M_{ab}^{(i)} = 0) \\ 0 & (M_{ab}^{(i)} < 0) \end{cases}, \quad (23)$$

where  $\tilde{z}_i \equiv z_i + \frac{\zeta_{ab}^{(i)}}{M_{ab}^{(i)}}$  and

$$I_{ab}^{(i)} = \begin{cases} 1, \dots, |M_{ab}^{(i)}| & (M_{ab}^{(i)} > 0) \\ 0 & (M_{ab}^{(i)} = 0) \\ \text{no solution} & (M_{ab}^{(i)} < 0) \end{cases}. \quad (24)$$

Due to the effect of chirality projection by fluxes, no zero-mode appears for  $M_{ab}^{(i)} < 0$ . On the other hand, for  $M_{ab}^{(i)} > 0$ , there appear  $M_{ab}^{(i)}$  zero-modes, and these zero-modes are labeled by the index  $I_{ab}^{(i)}$ . In Eq. (23), the wavefunction profile  $\Theta^{I, M}(z)$  is determined as

$$\Theta^{I, M}(z) \equiv \mathcal{N}_M e^{\pi i \frac{\text{Im } z}{\text{Im } \tau} M z} \vartheta \left[ \frac{I}{M} \right] (Mz, M\tau),$$

where  $\vartheta$  represents the Jacobi theta-function:

$$\vartheta \left[ \begin{smallmatrix} a \\ b \end{smallmatrix} \right] (\nu, \tau) = \sum_{l \in \mathbf{Z}} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}.$$

The normalization constant  $\mathcal{N}_M$  is found as

$$\mathcal{N}_{M^{(i)}} = \left( \frac{2 \text{Im } \tau_i |M^{(i)}|}{(\mathcal{A}^{(i)})^2} \right)^{1/4},$$

from a normalization condition,

$$\int dz_i d\bar{z}_i \sqrt{\det g^{(i)}} f^I (f^J)^* = \delta^{IJ},$$

for  $I, J \neq 0$  or  $I = J = 0$ .

For  $i \neq j$ , on the other hand, from Eq. (22) we find

$$(f_j^{(i)})_{ab} = f_{ab}^{I^{(i)}} \equiv \begin{cases} 0 & (M_{ab}^{(i)} > 0) \\ (\mathcal{A}^{(i)})^{-1/2} & (M_{ab}^{(i)} = 0) \\ \left( \Theta^{I_{ab}^{(i)}, |M_{ab}^{(i)}|}(\tilde{z}_i) \right)^* & (M_{ab}^{(i)} < 0) \end{cases},$$

where  $\tilde{z}_i \equiv z_i - \frac{\zeta_{ab}^{(i)}}{|M_{ab}^{(i)}|}$ , instead of Eq. (23).

### 3.2 4D effective action for zero-modes

Due to the gauge symmetry breaking  $U(N) \rightarrow \prod_a U(N_a)$  with  $a = 1, \dots, \tilde{N}$  caused by the fluxes satisfying Eq. (20), off-diagonal elements  $(V^{n=0})_{ab}$  ( $a \neq b$ ) obtain mass terms in Eq. (11), while diagonal elements  $(V^{n=0})_{aa}$  do not. Then, we express the zero-modes  $(V^{n=0})_{aa}$ , which contain gauge fields for the unbroken gauge symmetry  $\prod_a U(N_a)$ , as

$$(V^{n=0})_{aa} \equiv V^a.$$

On the other hand, from Eq. (24), for  $\exists j \neq i$  with  $M_{ab}^{(j)} < 0$  and  $M_{ab}^{(i)} > 0$  we find the zero-mode field  $(\phi_j^{n=0})_{ab}$  degenerates with a total degeneracy  $N_{ab} = \prod_i |M_{ab}^{(i)}|$ , while  $(\phi_j^{n=0})_{ba}$  has no zero-mode solution, yielding 4D chiral generations in the  $ab$ -sector. The opposite is true for  $M_{ab}^{(j)} > 0$  and  $M_{ab}^{(i)} < 0$  yielding 4D chiral generations in the  $ba$ -sector. Therefore, we denote the zero-modes  $(\phi_j^{n=0})_{ab}$  with the degeneracy  $N_{ab}$  as

$$(\phi_j^{n=0})_{ab} \equiv g \phi_j^{\mathcal{I}_{ab}},$$

where  $\mathcal{I}_{ab} \equiv (I_{ab}^{(1)}, I_{ab}^{(2)}, I_{ab}^{(3)})$  labels the degeneracy (generations). We normalize  $\phi_j^{\mathcal{I}_{ab}}$  by the 10D YM coupling constant  $g$ .

The analytic expressions of zero-mode wavefunctions allow us to derive 4D effective action after substituting the mode expansion (13) and integrating 10D action (5) over 6D extra space coordinates  $z_i$  and  $\bar{z}^{\bar{i}}$ . The effective action for zero-mode chiral superfields  $\phi_j^{\mathcal{I}_{ab}}$  is then found as

$$S_{\text{eff}} = \int d^4x \left[ \int d^4\theta \mathcal{K}_{\text{eff}} + \left\{ \int d^2\theta \left( \sum_a \frac{1}{4g_a^2} W^{a,\alpha} W_\alpha^a + \mathcal{W}_{\text{eff}} \right) + \text{h.c.} \right\} \right], \quad (25)$$

where

$$\begin{aligned} \mathcal{K}_{\text{eff}} &= \sum_{i,j} \sum_{a,b} \sum_{\mathcal{I}_{ab}} \tilde{Z}_{\mathcal{I}_{ab}}^{ij} \text{tr} \left[ \bar{\phi}_i^{\mathcal{I}_{ab}} e^{-V^a} \phi_j^{\mathcal{I}_{ab}} e^{V^b} \right], \\ \mathcal{W}_{\text{eff}} &= \sum_{i,j,k} \sum_{a,b,c} \sum_{\mathcal{I}_{ab}, \mathcal{I}_{bc}, \mathcal{I}_{ca}} \tilde{\lambda}_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}^{ijk} \text{tr} \left[ \phi_i^{\mathcal{I}_{ab}} \phi_j^{\mathcal{I}_{bc}} \phi_k^{\mathcal{I}_{ca}} \right], \end{aligned}$$

and

$$W_\alpha^a = -\frac{1}{4}\bar{D}\bar{D}e^{-V^a}D_\alpha e^{V^a}, \quad g_a = g\left(\prod_i \mathcal{A}^{(i)}\right)^{-1/2}.$$

The Kähler metric  $\tilde{Z}_{\mathcal{I}_{ab}}$  and the holomorphic Yukawa couplings  $\tilde{\lambda}_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}$  can be written as

$$\tilde{Z}_{\mathcal{I}_{ab}}^{\bar{i}j} = 2h^{\bar{i}j}, \quad (26)$$

$$\tilde{\lambda}_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}^{ijk} = -\frac{2g}{3}\epsilon^{ijk}e_i^ae_j^be_k^c\prod_r\tilde{\lambda}_{\mathcal{I}_{ab}^{(r)}\mathcal{I}_{bc}^{(r)}\mathcal{I}_{ca}^{(r)}}^{(r)}, \quad (27)$$

where  $r = 1, 2, 3$  and

$$\tilde{\lambda}_{\mathcal{I}_{ab}^{(r)}\mathcal{I}_{bc}^{(r)}\mathcal{I}_{ca}^{(r)}}^{(r)} = \int d^2z^r \sqrt{\det g^{(r)}} f_{ab}^{(r)} f_{bc}^{(r)} f_{ca}^{(r)}. \quad (28)$$

For  $a, b$  and  $c$  satisfying  $M_{ab}^{(r)}M_{bc}^{(r)}M_{ca}^{(r)} > 0$  (that is equivalent to  $M_{ac}^{(r)}M_{cb}^{(r)}M_{ba}^{(r)} < 0$ ), the overlap integral (28) is evaluated as

$$\tilde{\lambda}_{\mathcal{I}_{ab}^{(r)}\mathcal{I}_{bc}^{(r)}\mathcal{I}_{ca}^{(r)}}^{(r)} = \begin{cases} \tilde{\lambda}_{ab,c}^{(r)} & (M_{ab}^{(r)} > 0) \\ \tilde{\lambda}_{bc,a}^{(r)} & (M_{bc}^{(r)} > 0) \\ \tilde{\lambda}_{ca,b}^{(r)} & (M_{ca}^{(r)} > 0) \end{cases}, \quad (29)$$

where

$$\begin{aligned} \tilde{\lambda}_{ab,c}^{(r)} &= \mathcal{N}_{M_{ab}^{(r)}}^{-1}\mathcal{N}_{M_{bc}^{(r)}}\mathcal{N}_{M_{ca}^{(r)}}\sum_{m=1}^{M_{ab}^{(r)}}\delta_{\mathcal{I}_{bc}^{(r)}+\mathcal{I}_{ca}^{(r)}-mM_{bc}^{(r)},\mathcal{I}_{ab}^{(r)}} \\ &\times \exp\left[\frac{\pi i}{\text{Im}\tau_r}\left(\frac{\bar{\zeta}_{ab}^{(r)}}{M_{ab}^{(r)}}\text{Im}\zeta_{ab}^{(r)}+\frac{\bar{\zeta}_{bc}^{(r)}}{M_{bc}^{(r)}}\text{Im}\zeta_{bc}^{(r)}+\frac{\bar{\zeta}_{ca}^{(r)}}{M_{ca}^{(r)}}\text{Im}\zeta_{ca}^{(r)}\right)\right] \\ &\times \vartheta\left[\frac{M_{bc}^{(r)}\mathcal{I}_{ca}^{(r)}-M_{ca}^{(r)}\mathcal{I}_{bc}^{(r)}+mM_{bc}^{(r)}M_{ca}^{(r)}}{M_{ab}^{(r)}M_{bc}^{(r)}M_{ca}^{(r)}},\frac{0}{0}\right]\left(\bar{\zeta}_{ca}^{(r)}M_{bc}^{(r)}-\bar{\zeta}_{bc}^{(r)}M_{ca}^{(r)},-\bar{\tau}_rM_{ab}^{(r)}M_{bc}^{(r)}M_{ca}^{(r)}\right). \end{aligned}$$

For  $a, b$  and  $c$  satisfying  $M_{ab}^{(r)}M_{bc}^{(r)}M_{ca}^{(r)} = 0$ , on the other hand, the integral (28) is given by  $\tilde{\lambda}_{\mathcal{I}_{ab}^{(r)}\mathcal{I}_{bc}^{(r)}\mathcal{I}_{ca}^{(r)}}^{(r)} = (\mathcal{A}^{(r)})^{-1/2}$  instead of Eq. (29).

## 4 Local supersymmetry and moduli multiplets

The above derivation of 4D effective action has been performed in a limit of global supersymmetry, because our starting point is 10D SYM theory. From both theoretical and phenomenological viewpoints, however, theories with a local supersymmetry are desirable. It is well known that

10D SYM theories can be embedded into supergravity. Actually, low energy effective theories of heterotic and type I superstrings as well as type II orientifold/D-brane models are categorized into such a supergravity-YM system.

The existence of local supersymmetry implies that SYM system is coupled to gravity. With nonvanishing gravitational interactions, theories in higher-dimensional spacetime in general yield more massless modes from gravitational fields which form supermultiplets in the 4D effective theories. Especially, massless modes originating from the extra-dimensional components of higher-dimensional tensor (such as 10D graviton fields) and vector fields are called moduli which form chiral multiplets in 4D  $\mathcal{N} = 1$  supersymmetry.

In the following, we assume that there exists a local supersymmetry at the starting point of our previous analysis. Then, we show how to recover the local supersymmetry in the 4D effective theory, and identify the dependence of the effective action on geometric (so-called closed string) moduli as well as dilaton superfields.

## 4.1 4D $\mathcal{N} = 1$ effective supergravity

The action for 4D  $\mathcal{N} = 1$  conformal supergravity [12] is generally written as<sup>5</sup>

$$S_{\mathcal{N}=1} = \int d^4x \sqrt{-g^C} \left[ -3 \int d^4\theta \bar{C} C e^{-K/3} + \left\{ \int d^2\theta \left( \frac{1}{4} \sum_a f_a W^{a,\alpha} W_\alpha^a + C^3 W \right) + \text{h.c.} \right\} \right], \quad (30)$$

where  $C$  is a compensator chiral superfield, whose lowest component  $C|_0 \equiv C|_{\theta=\bar{\theta}=0}$  in the  $\theta$  and  $\bar{\theta}$  expansion relates the 4D metric  $g_{\mu\nu}^C$  in Eq. (30) with the one in Einstein frame  $g_{\mu\nu}^E$  as  $g_{\mu\nu}^C = (C\bar{C}|_0)^{-1} e^{K|_0/3} g_{\mu\nu}^E$ . Here and hereafter we denote the lowest component of a (function of) superfield  $\Phi$  in the  $\theta$  and  $\bar{\theta}$  expansion as  $\Phi|_{\theta=\bar{\theta}=0} \equiv \Phi|_0$ . The action of Poincare supergravity in Einstein frame is obtained [14] by a dilatation gauge fixing  $C|_0 = e^{K|_0/6}$  in the conformal supergravity action (30). Here and hereafter, we work in a unit that the 4D Planck scale is unity.

When we consider a situation that our starting 10D SYM theory is embedded in 10D supergravity, it is important to notify that there exists a scalar field  $\phi_{10}$  called dilaton in 10D supergravity-YM system, and the YM gauge coupling  $g$  is determined by a vacuum expectation value of 10D dilaton field  $\phi_{10}$  as

$$g = e^{\langle \phi_{10} \rangle / 2}.$$

Furthermore, the 4D effective action (25) is written in a so-called string frame, which is obtained by Eq. (30) with a dilatation gauge fixing,

$$C|_0 = e^{-\phi_4} e^{K|_0/6}, \quad (31)$$

---

<sup>5</sup> The superspace integrals  $\int d^4\theta \dots$  and  $\int d^2\theta \dots$  in Eq. (30) are a kind of simplified expressions for a notational convenience, which should be interpreted as  $D$ -term formula  $[\dots]_D$  and  $F$ -term formula  $[\dots]_F$  of superconformal tensor calculus [13], respectively.

where  $\phi_4$  is a zero-mode of  $\phi_{10}$ , which we call a 4D dilaton field. The VEV of  $\phi_4$  satisfies

$$e^{-2\langle\phi_4\rangle} = e^{-2\langle\phi_{10}\rangle} \prod_i \mathcal{A}^{(i)} = g^{-4} \prod_i \mathcal{A}^{(i)}. \quad (32)$$

The YM-field independent part of the Kähler potential  $K$  in the action (30), that we denote  $K^{(0)}$ , is well known as

$$K^{(0)} = \ln \frac{g^8}{2^7 (\prod_i \mathcal{A}_i)^2 \prod_i \text{Im } \tau_i},$$

determined from a coefficient of Einstein-Hilbert term in dimensionally reduced 4D effective supergravity action.

In the string frame (31), we expand the action (30) in powers of YM-fields and compare the corresponding terms in the power series to those in globally supersymmetric effective action (25). In this way, we determine the Kähler potential  $K$ , the superpotential  $W$  and the gauge kinetic function  $f_a$  in the 4D effective supergravity action (30) as

$$K = K^{(0)} + \sum_{i,j} \sum_{a,b} \sum_{\mathcal{I}_{ab}} Z_{\mathcal{I}_{ab}}^{\bar{i}j} \text{tr} \left[ \bar{\phi}_i^{\mathcal{I}_{ab}} e^{-V^a} \phi_j^{\mathcal{I}_{ab}} e^{V^b} \right], \quad (33)$$

$$W = \sum_{i,j,k} \sum_{a,b,c} \sum_{\mathcal{I}_{ab}, \mathcal{I}_{bc}, \mathcal{I}_{ca}} \lambda_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}^{ijk} \text{tr} \left[ \phi_i^{\mathcal{I}_{ab}} \phi_j^{\mathcal{I}_{bc}} \phi_k^{\mathcal{I}_{ca}} \right], \quad (34)$$

$$f_a = \frac{1}{g^2} \prod_i \mathcal{A}^{(i)}, \quad (35)$$

where the Kähler metric  $Z_{\mathcal{I}_{ab}}^{\bar{i}j}$  and the holomorphic Yukawa couplings  $\lambda_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}^{ijk}$  are given by

$$Z_{\mathcal{I}_{ab}}^{\bar{i}j} = e^{2\langle\phi_4\rangle} \tilde{Z}_{\mathcal{I}_{ab}}^{\bar{i}j}, \quad (36)$$

$$\lambda_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}^{ijk} = e^{3\langle\phi_4\rangle} e^{-K^{(0)}/2} \tilde{\lambda}_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}^{ijk}, \quad (37)$$

and  $\tilde{Z}_{\mathcal{I}_{ab}}^{\bar{i}j}$ ,  $\tilde{\lambda}_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}^{ijk}$  are given in Eqs. (26) and (27), respectively. Note that the VEV of 4D dilaton field  $\langle\phi_4\rangle$  in Eqs. (36) and (37) is related to  $g$  and  $\mathcal{A}^{(i)}$  as shown in Eq. (32).

## 4.2 Moduli dependence

As mentioned above, the YM gauge coupling constant  $g$  is determined by a VEV of 10D dilaton field  $\phi_{10}$  which is a dynamical scalar field. Furthermore, the geometric (torus) parameters  $R_i$  and  $\tau_i$  in the metric (2) should be considered as VEVs of dynamical fields originating from 10D metric  $G_{MN}$ . The zero-modes of these dynamical fields are called moduli. In the case of toroidal compactification, there exist Kähler moduli and complex structure moduli whose VEVs determine  $R_i$  and  $\tau_i$ , respectively, in addition to the zero-mode  $\phi_4$  of 10D dilaton field  $\phi_{10}$ .

We assume (at least  $\mathcal{N} = 1$ ) supersymmetric vacuum configurations (8) in this paper, and then geometric moduli as well as  $\phi_4$  form  $\mathcal{N} = 1$  supermultiplets described by chiral

superfields. Because these moduli fields are singlets under gauge transformations of SYM sector, they do not feel magnetic fluxes. Therefore, the formation of moduli supermultiplets is the same one as a pure toroidal case without magnetic fluxes, where we usually denote the dilaton, Kähler and complex-structure moduli chiral superfields as  $S$ ,  $T_i$  and  $U_i$ , respectively. A suitable identification of the VEVs of these fields are known [2] as

$$\text{Re} \langle S \rangle|_0 \equiv g^{-2} \prod_i \mathcal{A}^{(i)}, \quad \text{Re} \langle T_i \rangle|_0 \equiv g^{-2} \mathcal{A}^{(i)}, \quad \langle U_i \rangle|_0 \equiv i\bar{\tau}_i. \quad (38)$$

An important task here is to identify the dependence of 4D  $\mathcal{N} = 1$  effective supergravity action (30) on the dilaton and moduli chiral superfields  $S$ ,  $T_i$  and  $U_i$ . The Kähler and superpotential as well as the gauge kinetic functions in the action (30) are shown in Eqs. (33), (34) and (35), which are written in terms of VEVs of moduli, namely, parameters  $g$ ,  $R_i$  and  $\tau_i$ . Correct combinations of these parameters should be promoted to moduli chiral superfields  $S$ ,  $T_i$  and  $U_i$  following the relations (38) up to a certain parameter-dependent rescaling of YM superfields  $V^a$  and  $\phi_i^{\mathcal{I}ab}$  in the action (30). The rescaling is performed as

$$V^a \rightarrow V^a, \quad \phi_i^{\mathcal{I}ab} \rightarrow \alpha_{ab}^{(i)} \phi_i^{\mathcal{I}ab}, \quad (39)$$

where, for  $\prod_r M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)} \neq 0$ ,

$$\alpha_{ab}^{(i)} = \frac{1}{g\sqrt{2\text{Im}\tau_i}} \left( \prod_r \frac{\mathcal{A}^{(r)}}{\sqrt{2\text{Im}\tau_r}} \right)^{1/2} \exp \left[ - \sum_r \frac{\pi i}{\text{Im}\tau_r} \frac{\bar{\zeta}_{ab}^{(r)}}{M_{ab}^{(r)}} \text{Im}\zeta_{ab}^{(r)} \right] \left( \frac{|M_{ab}^{(i)}|}{\prod_{r \neq i} |M_{ab}^{(r)}|} \right)^{1/4} \quad (40)$$

so that all the parameters  $g$ ,  $R_i$  and  $\tau_i$  can be promoted to superfields  $S$ ,  $T_i$  and  $U_i$  through Eq. (38) with a proper holomorphicity in the superspace action. In a case with some vanishing fluxes,  $\prod_r M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)} = 0$ , the way of rescaling is shown in Appendix A.

After the above rescaling and promotion for  $\prod_r M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)} \neq 0$ , we find moduli dependence of the YM-field independent part of the Kähler potential  $K^{(0)}$ , the Kähler metric  $Z_{\mathcal{I}ab}^{\bar{i}j}$ , the holomorphic Yukawa couplings  $\lambda_{\mathcal{I}ab\mathcal{I}bc\mathcal{I}ca}^{ijk}$  and the gauge kinetic functions  $f_a$  as

$$K^{(0)}(S, T, U) = -\ln(S + \bar{S}) - \sum_r \ln(T_r + \bar{T}_r) - \sum_r \ln(U_r + \bar{U}_r), \quad (41)$$

$$\begin{aligned} Z_{\mathcal{I}ab}^{\bar{i}j}(S, T, U) &= \delta^{\bar{i}j} \left( \frac{T_j + \bar{T}_{\bar{j}}}{2} \right)^{-1} \left( \prod_r \frac{U_r + \bar{U}_{\bar{r}}}{2} \right)^{-1/2} \\ &\quad \times \frac{1}{2^{5/2}} \left( \frac{|M_{ab}^{(j)}|}{\prod_{r \neq j} |M_{ab}^{(r)}|} \right)^{1/2} \exp \left[ - \sum_r \frac{4\pi}{U_r + \bar{U}_{\bar{r}}} \frac{(\text{Im}\zeta_{ab}^{(r)})^2}{M_{ab}^{(r)}} \right], \end{aligned} \quad (42)$$

$$\lambda_{\mathcal{I}ab\mathcal{I}bc\mathcal{I}ca}^{ijk}(U) = -\frac{1}{3} \epsilon^{ijk} \delta_i^i \delta_j^j \delta_k^k \prod_r \lambda_{I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}}^{(r)}(U), \quad (43)$$

$$f_a(S) = S. \quad (44)$$

For  $a$ ,  $b$  and  $c$  satisfying  $M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)} > 0$  (that is equivalent to  $M_{ac}^{(r)} M_{cb}^{(r)} M_{ba}^{(r)} < 0$ ), the function  $\lambda_{I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}}^{(r)}(U)$  in Eq. (43) is evaluated as

$$\lambda_{I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}}^{(r)}(U) = \begin{cases} \lambda_{ab,c}^{(r)}(U) & (M_{ab}^{(r)} > 0) \\ \lambda_{bc,a}^{(r)}(U) & (M_{bc}^{(r)} > 0) \\ \lambda_{ca,b}^{(r)}(U) & (M_{ca}^{(r)} > 0) \end{cases}, \quad (45)$$

where

$$\begin{aligned} \lambda_{ab,c}^{(r)}(U) &= \sum_{m=1}^{M_{ab}^{(r)}} \delta_{I_{bc}^{(r)} + I_{ca}^{(r)} - m M_{bc}^{(r)}, I_{ab}^{(r)}} \\ &\times \vartheta \left[ \frac{M_{bc}^{(r)} I_{ca}^{(r)} - M_{ca}^{(r)} I_{bc}^{(r)} + m M_{bc}^{(r)} M_{ca}^{(r)}}{M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)}} \right] \left( \bar{\zeta}_{ca}^{(r)} M_{bc}^{(r)} - \bar{\zeta}_{bc}^{(r)} M_{ca}^{(r)}, i U_r M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)} \right). \end{aligned}$$

In a case with some vanishing fluxes,  $\prod_r M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)} = 0$ , the expression of Kähler metric is shown in Appendix A.

Note that these functions of moduli (41)-(44) appear in the action (30) with the Kähler potential (33) and the superpotential (34) where the YM fields  $V^a$  and  $\phi_i^{\mathcal{I}ab}$  represent those after the rescaling (40). These results (41)-(44) are consistent with those obtained in different ways [2, 15]. A systematic formulation of 4D effective theory respecting  $\mathcal{N} = 1$  superspace structure presented here will be easily adopted to more general systems of magnetized SYM theories and D-branes.

## 5 An example of model building

In this section, we indicate a possible direction of phenomenological model building based on our formulation. Starting from 10D  $U(N)$  SYM theory with  $N = 8$ , we assume magnetic fluxes yielding  $\tilde{N} = 3$  and  $(N_1, N_2, N_3) \equiv (N_C, N_L, N_R) = (4, 2, 2)$  in Eq. (20) that break YM symmetry as  $U(8) \rightarrow U(4)_C \times U(2)_L \times U(2)_R$ . We consider the case that this is further broken down to  $U(3)_C \times U(1)_{C'} \times U(2)_L \times U(1)_{R'} \times U(1)_{R''}$  by Wilson-lines yielding  $\tilde{N} = 5$  and  $(N_1, N_2, N_3, N_4, N_5) \equiv (N_C, N_{C'}, N_L, N_{R'}, N_{R''}) = (3, 1, 2, 1, 1)$ . The situation is realized [16] by the following magnetic fluxes and Wilson-lines for  $r = 1, 2, 3$ :

$$\begin{aligned} F_{2+2r, 3+2r} &= 2\pi \begin{pmatrix} M_C^{(r)} \mathbf{1}_4 & & \\ & M_L^{(r)} \mathbf{1}_2 & \\ & & M_R^{(r)} \mathbf{1}_2 \end{pmatrix}, \\ \zeta_r &= \begin{pmatrix} \zeta_C^{(r)} \mathbf{1}_3 & & & & \\ & \zeta_{C'}^{(r)} & & & \\ & & \zeta_L^{(r)} \mathbf{1}_2 & & \\ & & & \zeta_{R'}^{(r)} & \\ & & & & \zeta_{R''}^{(r)} \end{pmatrix}, \end{aligned}$$



where  $\mathbf{1}_N$  is a  $N \times N$  unit matrix, and all the nonvanishing entries take different values from each other.

We embed the gauge symmetries  $SU(3)_C$  and  $SU(2)_L$  of the standard model into the above unbroken gauge groups as  $SU(3)_C \subset U(3)_C$  and  $SU(2)_L \subset U(2)_L$ . Then, in order to obtain three generations of quarks and leptons from the zero-mode degeneracy (24) and full-rank Yukawa matrices from the 10D gauge interaction, the magnetic fluxes are determined, e.g., as

$$\begin{aligned} (M_C^{(1)}, M_L^{(1)}, M_R^{(1)}) &= (0, +3, -3), \\ (M_C^{(2)}, M_L^{(2)}, M_R^{(2)}) &= (0, -1, 0), \\ (M_C^{(3)}, M_L^{(3)}, M_R^{(3)}) &= (0, 0, +1), \end{aligned} \quad (46)$$

which correspond to

$$\begin{aligned} M_C^{(1)} - M_L^{(1)} &= -3, & M_L^{(1)} - M_R^{(1)} &= +6, & M_R^{(1)} - M_C^{(1)} &= -3, \\ M_C^{(2)} - M_L^{(2)} &= +1, & M_L^{(2)} - M_R^{(2)} &= -1, & M_R^{(2)} - M_C^{(2)} &= 0, \\ M_C^{(3)} - M_L^{(3)} &= 0, & M_L^{(3)} - M_R^{(3)} &= -1, & M_R^{(3)} - M_C^{(3)} &= +1. \end{aligned} \quad (47)$$

In this case, supersymmetry conditions (18) and (19) are satisfied by

$$\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = \mathcal{A}^{(1)}/\mathcal{A}^{(3)} = 3. \quad (48)$$

In this model, chiral superfields  $Q$ ,  $U$ ,  $D$ ,  $L$ ,  $N$ ,  $E$ ,  $H_u$  and  $H_d$  carrying the left-handed quark ( $Q$ ), the right-handed up-type quark ( $U$ ), the right-handed down-type quark ( $D$ ), the left-handed lepton ( $L$ ), the right-handed neutrino ( $N$ ), the right-handed electron ( $E$ ), the up-type Higgs particle ( $H_u$ ) and the down-type Higgs particle ( $H_d$ ), respectively, are found in  $\phi_i^{\mathcal{I}ab}$  as

$$\phi_1^{\mathcal{I}ab} = \left( \begin{array}{cc|cc} \Omega_C^{(1)} & \Xi_{CC'}^{(1)} & 0 & \Xi_{CR'}^{(1)} & \Xi_{CR''}^{(1)} \\ \Xi_{C'C}^{(1)} & \Omega_{C'}^{(1)} & 0 & \Xi_{C'R'}^{(1)} & \Xi_{C'R''}^{(1)} \\ \hline \Xi_{LC}^{(1)} & \Xi_{LC'}^{(1)} & \Omega_L^{(1)} & H_u^K & H_d^K \\ \hline 0 & 0 & 0 & \Omega_{R'}^{(1)} & \Xi_{R'R''}^{(1)} \\ \hline 0 & 0 & 0 & \Xi_{R''R'}^{(1)} & \Omega_{R''}^{(1)} \end{array} \right), \quad (49)$$

$$\phi_2^{\mathcal{I}ab} = \left( \begin{array}{cc|cc} \Omega_C^{(2)} & \Xi_{CC'}^{(2)} & Q^I & 0 & 0 \\ \Xi_{C'C}^{(2)} & \Omega_{C'}^{(2)} & L^I & 0 & 0 \\ \hline 0 & 0 & \Omega_L^{(2)} & 0 & 0 \\ \hline 0 & 0 & 0 & \Omega_{R'}^{(2)} & \Xi_{R'R''}^{(2)} \\ \hline 0 & 0 & 0 & \Xi_{R''R'}^{(2)} & \Omega_{R''}^{(2)} \end{array} \right), \quad (50)$$

$$\phi_3^{\mathcal{I}ab} = \left( \begin{array}{cc|cc} \Omega_C^{(3)} & \Xi_{CC'}^{(3)} & 0 & 0 & 0 \\ \Xi_{C'C}^{(3)} & \Omega_{C'}^{(3)} & 0 & 0 & 0 \\ \hline 0 & 0 & \Omega_L^{(3)} & 0 & 0 \\ \hline U^J & N^J & 0 & \Omega_{R'}^{(3)} & \Xi_{R'R''}^{(3)} \\ \hline D^J & E^J & 0 & \Xi_{R''R'}^{(3)} & \Omega_{R''}^{(3)} \end{array} \right), \quad (51)$$

where the rows and columns of matrices correspond to  $a = 1, \dots, 5$  and  $b = 1, \dots, 5$ , respectively, and the indices  $I, J = 1, 2, 3$  and  $K = 1, \dots, 6$  label generations. Three generations of  $Q, U, D, L, N, E$  and six generations of  $H_u$  and  $H_d$  are generated by the fluxes (46). The Kähler metric and holomorphic Yukawa couplings for these superfields are easily derived from Eqs. (42) and (43).

Note that each of zero entries in the matrices (49), (50) and (51) represents eliminated components due to the effect of chirality projection caused by magnetic fluxes. However, because we require some vanishing fluxes in Eq. (47) in order to obtain three generations of quarks and leptons, there appear some massless exotic modes  $\Xi_{ab}^{(r)}$  as well as diagonal components  $\Omega_a^{(r)}$  (so-called open string moduli), all of which feel zero fluxes. Some of these modes can be eliminated if we consider certain orbifold projections on  $r = 2, 3$  tori, that is, a magnetized orbifold model [4]. More details of this model building and phenomenological features at a low energy will be reported in a separate paper [17].

## 6 Conclusion

We have presented 4D  $\mathcal{N} = 1$  superfield description of 10D SYM theories compactified on magnetized tori which preserve the  $\mathcal{N} = 1$  supersymmetry. Based on such a description, we have derived 4D effective action for massless zero-modes written in the  $\mathcal{N} = 1$  superspace. We further identified moduli dependence of the effective action by promoting the YM gauge coupling constant  $g$  and geometric parameters  $R_i$  and  $\tau_i$  to a dilaton, Kähler and complex-structure moduli superfields. The resulting effective supergravity action would be useful for building phenomenological models and for analyzing them systematically.

Although we have worked on 10D SYM theories in this paper, it is straightforward to adopt our formulation to SYM in lower-than-ten dimensional spacetime, in a similar way to the one suggested in Ref. [10] without magnetic fluxes. A local supersymmetry can be recovered in 4D effective theories<sup>6</sup> following the procedure presented in Sec. 4. Then, e.g., in type IIB orientifolds, our formulation will be applied not only to magnetized D9 branes (a class of which is T-dual to intersecting D6 branes in IIA side), but also to D5-D9 [19] and D3-D7 brane configurations with magnetic fluxes in extra dimensions. Lower-dimensional brane configurations may allow the introduction of supersymmetry-breaking branes sequestered from the visible sector, which coincide with flavor structures in the visible sector generated by magnetic fluxes.

The explicit moduli dependence of the superspace effective action also allows us to study a moduli stabilization and a supersymmetry breaking at a Minkowski minimum [20] based on SYM theories, by minimizing the moduli and hidden-sector potential generated by some combinations [21] of nonperturbative effects and a dynamical supersymmetry breaking [22]. Then, it would be possible to determine explicit forms of soft terms in the visible sector generated by moduli-mediated supersymmetry breaking (or mixed modulus-anomaly mediation [23]). In such models, brane configurations in the higher-dimensional spacetime might be detected by

---

<sup>6</sup> The corresponding supergravity actions in the original higher-dimensional spacetime could be also written in  $\mathcal{N} = 1$  superspace as shown in the case of five dimensions [18].

measuring supersymmetric flavor structures at a low energy. The formulation of 4D effective action presented here would be suitable for such analyses.

## Acknowledgement

The work of H. A. was supported by the Waseda University Grant for Special Research Projects No.2011B-177. The work of T. K. is supported in part by a Grant-in-Aid for Scientific Research No. 20540266 and the Grant-in-Aid for the Global COE Program “The Next Generation of Physics, Spun from Universality and Emergence” from the Ministry of Education, Culture, Sports, Science and Technology of Japan. The work of H. O. is supported by the JSPS Grant-in-Aid for Scientific Research (S) No. 22224003. H. A. and T. K. thank the Yukawa Institute for Theoretical Physics at Kyoto University. Discussions during the YITP workshop ”Summer Institute 2011” were useful to complete this work.

## A Rescaling fields with some vanishing fluxes

In typical cases with some vanishing fluxes,  $\prod_r M_{ab}^{(r)} M_{bc}^{(r)} M_{ca}^{(r)} = 0$ , the way of rescaling (39) and the corresponding Kähler metric are found as

$$\begin{aligned} \alpha_{ab}^{(i)} &= \frac{1}{g\sqrt{2\operatorname{Im}\tau_i}} \left( \prod_r \frac{\mathcal{A}^{(r)}}{\sqrt{2\operatorname{Im}\tau_r}} \right)^{1/2} \exp \left[ - \sum_{r \neq k} \frac{\pi i}{\operatorname{Im}\tau_r} \frac{\bar{\zeta}_{ab}^{(r)}}{M_{ab}^{(r)}} \operatorname{Im}\zeta_{ab}^{(r)} \right] \\ &\times \left( \frac{|M_{ab}^{(i)}|}{\prod_{r \neq i} |M_{ab}^{(r)}|} \right)^{1/4} \left( 2\operatorname{Im}\tau_k |M_{ab}^{(k)}| \right)^{1/4}, \end{aligned}$$

and

$$\begin{aligned} Z_{\mathcal{I}_{ab}}^{\bar{i}j}(S, T, U) &= \delta^{\bar{i}j} \left( \frac{T_j + \bar{T}_j}{2} \right)^{-1} \left( \prod_{r \neq k} \frac{U_r + \bar{U}_r}{2} \right)^{-1/2} \\ &\times \frac{1}{2^2} \left( \frac{|M_{ab}^{(j)}|}{\prod_{r \neq j} |M_{ab}^{(r)}|} \right)^{1/2} |M_{ab}^{(k)}|^{1/2} \exp \left[ - \sum_{r \neq k} \frac{4\pi}{U_r + \bar{U}_r} \frac{(\operatorname{Im}\zeta_{ab}^{(r)})^2}{M_{ab}^{(r)}} \right], \end{aligned}$$

for  $M_{ab}^{(k)} = 0$  and  $\exists k \neq i$  with others nonvanishing, and

$$\begin{aligned} \alpha_{ab}^{(i)} &= \frac{1}{g\sqrt{2\operatorname{Im}\tau_i}} \left( \prod_r \frac{\mathcal{A}^{(r)}}{\sqrt{2\operatorname{Im}\tau_r}} \right)^{1/2} \exp \left[ - \sum_{r \neq k} \frac{\pi i}{\operatorname{Im}\tau_r} \frac{\bar{\zeta}_{ab}^{(r)}}{M_{ab}^{(r)}} \operatorname{Im}\zeta_{ab}^{(r)} \right] \\ &\times \left( \frac{|M_{ab}^{(i)}|}{\prod_{r \neq i} |M_{ab}^{(r)}|} \right)^{1/4} |M_{ab}^{(k)}|^{(-1)^{\delta_{ik}}/4}, \end{aligned}$$

and

$$Z_{\mathcal{I}_{ab}}^{\bar{i}j}(S, T, U) = \delta^{\bar{i}j} \left( \frac{T_j + \bar{T}_{\bar{j}}}{2} \right)^{-1} \left( \prod_r \frac{U_r + \bar{U}_{\bar{r}}}{2} \right)^{-1/2} \\ \times \frac{1}{2^{5/2}} \left( \frac{|M_{ab}^{(j)}|}{\prod_{r \neq j} |M_{ab}^{(r)}|} \right)^{1/2} |M_{ab}^{(k)}|^{(-1)^{\delta_{jk}}/2} \exp \left[ - \sum_{r \neq k} \frac{4\pi}{U_r + \bar{U}_{\bar{r}}} \frac{(\text{Im } \zeta_{ab}^{(r)})^2}{M_{ab}^{(r)}} \right],$$

for  $M_{bc}^{(k)} = 0$  or  $M_{ca}^{(k)} = 0$  and  $\exists k$  with others nonvanishing, and so on.

## References

- [1] L. E. Ibanez and A. M. Uranga, “String theory and particle physics: An introduction to string phenomenology,” Cambridge University Press (2012).
- [2] D. Cremades, L. E. Ibanez and F. Marchesano, JHEP **0405** (2004) 079 [hep-th/0404229].
- [3] J. P. Conlon, A. Maharana and F. Quevedo, JHEP **0809**, 104 (2008) [arXiv:0807.0789 [hep-th]]; F. Marchesano, P. McGuirk and G. Shiu, JHEP **0904**, 095 (2009) [arXiv:0812.2247 [hep-th]]; P. G. Camara and F. Marchesano, JHEP **0910**, 017 (2009) [arXiv:0906.3033 [hep-th]].
- [4] H. Abe, T. Kobayashi and H. Ohki, JHEP **0809** (2008) 043 [arXiv:0806.4748 [hep-th]]; H. Abe, K. -S. Choi, T. Kobayashi and H. Ohki, Nucl. Phys. B **814** (2009) 265 [arXiv:0812.3534 [hep-th]].
- [5] K. S. Choi, T. Kobayashi, R. Maruyama, M. Murata, Y. Nakai, H. Ohki and M. Sakai, Eur. Phys. J. C **67**, 273 (2010) [arXiv:0908.0395 [hep-ph]]; T. Kobayashi, R. Maruyama, M. Murata, H. Ohki and M. Sakai, JHEP **1005**, 050 (2010) [arXiv:1002.2828 [hep-ph]].
- [6] H. Abe, K. -S. Choi, T. Kobayashi and H. Ohki, Nucl. Phys. B **820** (2009) 317 [arXiv:0904.2631 [hep-ph]]; Phys. Rev. D **80** (2009) 126006 [arXiv:0907.5274 [hep-th]]; Phys. Rev. D **81** (2010) 126003 [arXiv:1001.1788 [hep-th]].
- [7] T. Kobayashi, S. Raby and R. J. Zhang, Nucl. Phys. B **704**, 3 (2005) [arXiv:hep-ph/0409098]; T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B **768**, 135 (2007) [arXiv:hep-ph/0611020]; P. Ko, T. Kobayashi, J. h. Park and S. Raby, Phys. Rev. D **76**, 035005 (2007) [Erratum-ibid. D **76**, 059901 (2007)] [arXiv:0704.2807 [hep-ph]].
- [8] R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, JHEP **0010** (2000) 006 [hep-th/0007024].
- [9] N. Marcus, A. Sagnotti and W. Siegel, Nucl. Phys. B **224** (1983) 159.

- [10] N. Arkani-Hamed, T. Gregoire and J. G. Wacker, JHEP **0203** (2002) 055 [hep-th/0101233].
- [11] J. Wess and J. Bagger, Princeton, USA: Univ. Pr. (1992) 259 p
- [12] M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, Phys. Rev. Lett. **39** (1977) 1109.
- [13] T. Kugo and S. Uehara, Nucl. Phys. B **226** (1983) 49.
- [14] M. Kaku and P. K. Townsend, Phys. Lett. B **76** (1978) 54.
- [15] P. Di Vecchia, A. Liccardo, R. Marotta and F. Pezzella, JHEP **0903** (2009) 029 [arXiv:0810.5509 [hep-th]].
- [16] H. Ohki, arXiv:1003.5194 [hep-th].
- [17] H. Abe, T. Kobayashi, H. Ohki, A. Oikawa and K. Sumita, in progress.
- [18] F. Paccetti Correia, M. G. Schmidt and Z. Tavartkiladze, Nucl. Phys. B **709** (2005) 141 [hep-th/0408138]; H. Abe and Y. Sakamura, JHEP **0410** (2004) 013 [hep-th/0408224].
- [19] P. Di Vecchia, R. Marotta, I. Pesando and F. Pezzella, J. Phys. A **44**, 245401 (2011) [arXiv:1101.0120 [hep-th]].
- [20] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D **68** (2003) 046005 [hep-th/0301240].
- [21] E. Dudas, C. Papineau and S. Pokorski, JHEP **0702** (2007) 028 [hep-th/0610297]; H. Abe, T. Higaki, T. Kobayashi and Y. Omura, Phys. Rev. D **75** (2007) 025019 [hep-th/0611024].
- [22] K. A. Intriligator, N. Seiberg and D. Shih, JHEP **0604** (2006) 021 [hep-th/0602239].
- [23] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP **0411** (2004) 076 [hep-th/0411066]; K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B **718** (2005) 113 [hep-th/0503216].